1882MNRAS..42..267N

Note on a Term in the Perturbations of the Moon, due to the Action of Mars. By E. Neison, Esq.

When at work in the beginning of 1877 on the theory of the perturbations of the Moon arising from the disturbing action of the planets, I was lead to approximately calculate the value of a number of terms of long period. One of these came out with a coefficient of several seconds of arc, it being the term with the argument

$$l - 24\,l'' + 20\,l''',$$

where l, l'', l''', denote the mean longitudes of the Moon, Earth and *Mars*. It so happened that this term closely represented in both amount and period the correction required to bring theory and observation into accord during the long period 1765–1851, as indicated by the residuals given by Sir George Airy in his reduction of the Greenwich Observations (see *M. N.* Nov. 1873). This led me to communicate this provisional result to the Society, with the caution that, from "the nature of the new term, it is obvious that the value which has been deduced for it in the present Paper is at best merely provisional. It will require most careful revision, and the higher powers of the disturbing forces must be taken into account. . . . . The existence of this term is so strongly supported by observation, from the manner in which it removes the discordance between theory and observation, that I have thought it better to publish it in its present form rather than defer this for the considerable period which may elapse before I am able to verify these results" (*M. N.* vol. xxxviii. p. 53).

At that time I had not discovered how much these outstanding residuals required correction for existing errors in the theory before they properly represented the difference between observation and theory, or I should not have attached such a great weight to this agreement as a confirmation of the accuracy of my rough calculation.

The value of the coefficient of this term entirely depends on the exact value of the very small difference of the sum of two long series of very large quantities of different signs, so that if each term of these series be not calculated with the greatest exactness, the difference between the sums of these two series may be entirely incorrect. In my calculations I had employed quantities extending to five places, which seemed amply sufficient; but in the course of the following year I found that this was not so, and that, in consequence, the difference between the sums of these two long series was far too great, so that the value found for the coefficient of this new term of long period was far too large. This fact I mentioned in my paper on "Hansen's Terms of Long Period" (*M. N.* March 1878, p. 269).

Downloaded from http://mnras.oxfordjournals.org/ at Carleton University on June 20, 2015

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System

1882MNRAS..42..267N

Since then I have more accurately computed the value of this part of this term, and find its true value to be only a small fraction of a second, instead of several seconds, and the calculation forms a portion of the second part of my still unpublished Memoir on the "Perturbations of the Moon due to the Action of the Planets."

Lately I have received from M. Constantin Gogau an elaborate investigation of this portion of the complete value of this term—that is, the portion arising from the direct action of *Mars* on the Moon as disturbed by the Sun. Adopting the method employed by Delaunay for determining the value of the similar inequalities computed by Hansen, M. Gogau has investigated with great care the exact analytical expression for the coefficient of this term of long period as far as the third power of the eccentricities and inclinations of the Earth and *Mars*, and the second power of the ratio of the mean motions of the Earth and Moon. Then, by reducing these analytical results to numbers, he arrives at the conclusion that the value of the coefficient of this term is so small a fraction of a second of arc that the term must be considered quite insensible.

M. Gogau seems to have thought that my provisional calculation was intended for a complete investigation, for he points out that I have neglected a number of small quantities which ought to be included in a complete determination of the value of this coefficient. This is, of course, obvious; but the omission was intentional, for, as I state in my Paper, I omitted these terms because they could not yield any sensible portion of the coefficient. M. Gogau also lays much stress on the fact that I have omitted to take into account certain terms which are independent of m, the ratio of the mean motions of the Earth and Moon; but these terms, though they do not implicitly involve m, are none the less dependent on the disturbing force of the Sun, and cannot therefore be properly included in any investigation in which this disturbing force is neglected, as I explicitly stated was the case in the present instance. As I have pointed out elsewhere (*M. N.* March 1878) these terms depending on the solar disturbing force cannot be of any material importance in these inequalities of long period, but can only form a very small fraction of the value which is independent of them. Their consideration gives rise to an enormous increase in the labour of the investigation, but, as M. Gogau's results clearly show, their omission would not affect the conclusions in any material degree.

In so far as they extend, M. Gogau's results appear accurate. But they do not form a satisfactory determination of even this part of the complete coefficient, for they are not sufficiently extended. The terms depending on the fifth and seventh powers and products of the eccentricities of *Mars* and the Earth are capable of yielding far larger quantities than found to result from the third powers, and until it be shown that the mutual destruction of terms is as complete amongst the higher powers

Downloaded from http://mnras.oxfordjournals.org/ at Carleton University on June 20, 2015

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System